A Penalty Scheme for Mitigating Uninstructed Deviation of Generation Outputs from Variable Renewables in a Distribution Market

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***Abstract* – With rapid growth of distributed renewable generation, the establishment of electricity distribution markets has attracted widespread concerns. Different from existing transmission grid-scale electricity markets, an electricity distribution market is featured by numerous small-scale prosumers, and zero marginal cost and intermittency of renewable generation units. Against this background, this paper first extends an average pricing market (APM) mechanism for pricing renewable generation outputs with zero marginal cost in the distribution network concerned. Then, to mitigate the uninstructed volatility of renewable generation outputs and power demand, a penalty scheme is proposed for deviations between the real-time demand/output and market cleared bid/offer, with frequency regulation service (FRS) from energy storage systems (ESSs) considered. It is proved that the market volatility can be well controlled within an expected limit through properly setting the penalty prices for load demand and generation output fluctuations. Also, with this mechanism a non-negative market surplus could always be attained. Case studies are carried out to demonstrate the feasibility and efficiency of the proposed distribution market mechanism and penalty scheme.**

***Index Terms*—electricity distribution market, renewable generation, zero marginal cost, energy storage system (ESS), frequency regulation service (FRS), penalty scheme.**

NOMENCLATURE

Indices and sets

i	Index of consumers, $i \in N$
j	Index of producers, $j \in M$
s	Index of time slots
N / M	Set of consumers / produces in the market

Parameters

r_i^{b} / r_j^{s}	Bidding / offering prices of the i^{th} consumer / j^{th} producer
p_i^{b} / p_j^{s}	Bidding load / output of the i^{th} consumer / j^{th} producer
Δt	Internal of transaction in a distribution market
$\bar{r}$	Market clearing price (MCP)
r^{retail}	Incumbent electricity retail price
r^{feed}	Feed-in tariff in a distribution system
$r^{\mathrm{c,ESS}}$ / r^{mil}	Capacity / mileage price of ESS FRS
$p_{i,t}^{\mathrm{b,acl}}$ / $p_{j,t}^{\mathrm{s,acl}}$	Actual load demand of the i^{th} consumer / actual output of the j^{th} producer at time t
$r^{\mathrm{b,pnl,c}}$ / $r^{\mathrm{s,pnl,c}}$	Penalty price on the maximum power rate deviation of load demand / generation output
$r^{\mathrm{b,pnl,mil}}$/$r^{\mathrm{s,pnl,mil}}$	Penalty price on the deviation mileage of load demand / generation output
$r^{\mathrm{b,pnl,e}}$ / $r^{\mathrm{s,pnl,e}}$	Penalty price on the difference of electricity quantity between the real-time and market cleared load demand / generation output
$p_i^{\mathrm{b,ESS}}$ / $p_j^{\mathrm{s,ESS}}$	Capacity of self-equipped ESS for the i^{th} consumer / the j^{th} producer
N^{disp}	Total number of recorded demand/output data during Δt

Functions and Variables

p_i^{cb} / p_j^{cs}	Dispatched load / output of the i^{th} consumer /j^{th} producer
$p_i^{\mathrm{b,ESS,c}}$ / $p_j^{\mathrm{s,ESS,c}}$	Purchased ESS capacity for FRS by the i^{th} consumer / j^{th} producer
$p_i^{\mathrm{b,ESS,rsv}}$/$p_j^{\mathrm{s,ESS,rsv}}$	Capacity of self-equipped ESS for arbitrage for the i^{th} consumer / j^{th} producer
$p_i^{\mathrm{b,ESS,mkt}}$/$p_j^{\mathrm{s,ESS,mkt}}$	Capacity of self-equipped ESS for selling FRS to others for the i^{th} consumer / j^{th} producer
$p_i^{\mathrm{b,ESS,reg}}$/$p_j^{\mathrm{s,ESS,reg}}$	Capacity of self-equipped ESS for self-regulation for the i^{th} consumer / j^{th} producer
R_i^{b} / R_j^{s}	Total benefit of participating in the distribution market for the i^{th} consumer / j^{th} producer
$R_i^{\mathrm{b,ESS,rsv}}$/$R_j^{\mathrm{s,ESS,rsv}}$	Revenue of arbitrage for the i^{th} consumer / j^{th} producer
$R_i^{\mathrm{b,ESS,mkt}}$/$R_j^{\mathrm{s,ESS,mkt}}$	Revenue of selling FRS to others for the i^{th} consumer / j^{th} producer

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$R_i^{\text{b,ret}}$ / $R_j^{\text{s,ret}}$	Saving of money by participating in the distribution market for the i^{th} consumer / j^{th} producer
$C_i^{\text{b,ESS,FRS}}$/$C_j^{\text{s,ESS,FRS}}$	Cost of purchasing ESS FRS capacity and regulation mileage for the i^{th} consumer / j^{th} producer
$C_I^{\text{b,pnl,c}}$ / $C_j^{\text{s,pnl,c}}$,	Penalty on maximum power rate deviation for the i^{th} consumer / j^{th} producer
$C_i^{\text{b,pnl,mil}}$ / $C_j^{\text{s,pnl,mil}}$	Penalty on deviation mileage for the i^{th} consumer / j^{th} producer
$C_i^{\text{b,pnl,e}}$ / $C_j^{\text{s,pnl,e}}$	Penalty on the difference of electricity quantity for the i^{th} consumer / j^{th} producer

I. INTRODUCTION

TO help customers benefit from the installed distributed renewable generation facilities, a variety of trials and projects on peer-to-peer (P2P) electricity trading in the distribution side has been carried out in some countries. These projects usually aim to lower the retail price for consumers and increase the feed-in tariff for producers. Meanwhile, technical progresses in modern smart grids, such as the advent of the so-called smart home, numerous installations of intelligent metering devices, and integration of information and communication technologies (ICTs), will stimulate the establishment of a marketplace for transactions in a distribution system.

Electricity trading in a distribution market is different from that in a wholesale electricity market in the following aspects. First, participants in a distribution market include some energy prosumers (producers-and-consumers) equipped with small-scale renewable generators. The bid or offered transaction volume of electricity from each participant which can be a prosumer, a producer or a consumer is usually small. Secondly, the traded electricity is almost 100% from renewable generation in the electricity distribution market. Compared with traditional generation technologies, renewable generation is capital-intensive but has zero fuel cost [1]. Thirdly, the distribution market participants usually do not have sufficient capability or time to develop optimal bidding strategies through sophisticated computation; thus, they would prefer a set-and-forget method for setting bid parameters when participating in the electricity distribution market. Besides, prosumers in a distribution system are featured by random usage behaviours of electricity and uncertain renewable generation outputs; the distribution market mechanism thus should be compatible with uncertainties of electricity transactions.

There are some publications on electricity distribution markets already. In [2-6], an electricity distribution market is modelled as an intermediate entity between the wholesale electricity market and distribution network customers. Through communications between the independent system operator (ISO) and proactive customers, a distribution market operator (DMO) enables customers participate in the wholesale electricity market. The distribution locational marginal price (DLMP) is introduced for market settlement in [3, 4, 6], which is similar to the concept of locational marginal price (LMP) in the wholesale electricity market. Due to relatively high power losses, voltage volatilities, and phase imbalances in the distribution network, the determination of DLMP is a non-trivial issue. Therefore, in [7], a three-phase optimal power flow (OPF) based approach is developed to define and calculate DLMP. The typical decentralized electricity trading mechanisms in distribution systems are surveyed and reviewed in [8, 9], where key problems in the decentralized electricity transaction are also analysed. Besides, a framework of designing and simulating electric distribution systems and day-ahead electricity distribution markets in UK is studied in [10], with all generators assumed to be price-takers and offer at their marginal costs.

However, some aforementioned key features of transactions in the electricity distribution market are overlooked in these publications, and the true values of renewable energy generation cannot be well revealed, and the market operation efficiency will then be low. In particular, an average pricing market (APM) mechanism for renewable generators with zero marginal costs in an electricity distribution market is proposed in [11] with a double-sided bidding market mechanism established. However, the uncertainties associated with the market clearing outcomes due to variable demand behaviours and intermittent renewable generation outputs are not studied in [11]. Therefore, this paper further extended the electricity distribution market mechanism in [11] by proposing a penalty scheme for mitigating the uninstructed deviations of generation outputs from variable renewables and power demands in a distribution market. In a power system, load and generation schedule deviations could be instructed or uninstructed [12]. Because uninstructed deviation (which is not a response to dispatch instruction from the system operator) can adversely affect reliability and raise operation costs of a distribution system, establishing a penalty scheme for mitigating the negative impacts of uninstructed deviations is of great significance and is the focus of this paper. Hereafter, the deviation refers to the uninstructed one.

Since the value of energy storage systems (ESSs) to provide frequency regulation service (FRS) has been more and more recognized, the design of control strategies for ESS based FRS is studied by researchers. A finite-time leader–follower consensus algorithm is proposed in [13] to control the small-scale ESSs via sparse communication networks. In [14], an intelligent control strategy based on the adaptive dynamic programming is developed for frequency regulation in a microgrid with micro-turbine and ESS. Besides, a comprehensive survey about the connection requirements, design considerations, service characteristics and real-world implementation of grid-scale ESS for FRS in power systems is conducted in [15]. Meanwhile, some other publications studied the siting and sizing problems in power systems by minimizing their expected investment and operation costs, where ESSs make profits through spatial-temporal energy arbitrage instead of offering ancillary services [16-20]. FRS provided by an ESS in the distribution market is studied here when designing the penalty scheme, as prosumers with excessive ESS capacities are capable of participating in the ancillary service market.

The main contributions of this paper are summarized below.

First, the APM mechanism is extended from the energy market to the ancillary service market. While honesty bidding is proved to be a dominant strategy for participants in the APM mechanism, this paper further proves that the dominant strategy of participants under the APM mechanism is independent of the bidding quantities of demand/power generation. Thus, honesty bidding will continue to be the dominant strategy when the APM is adopted for a joint energy and ancillary service market, since the bidding strategies of participants in the energy market will not be impacted by the cost of purchasing FRS from ESSs.

Secondly, a penalty scheme is established to ensure the successful implementation of distribution market clearing outcomes, which is not studied in existing research. Under the proposed penalty scheme, it is proved that the market volatility can be controlled within an expected limit through properly setting the penalty prices. Besides, a non-negative market surplus can always be attained due to the complementariness of load demand and generation output fluctuations.

The rest of this paper is organized as follows. Section II introduces the clearing mechanism in an electricity distribution market. Then, a penalty scheme on the deviation of real-time demand and generation output is presented in Section III. Section IV presents and discusses case study results. Finally, the paper is concluded in Section V.

II. Electricity Distribution Market Clearing Mechanism

A. *Participation and Responsibility in a Distribution Market*

The distribution market is defined as a competitive platform in the distribution network that enables market activities for proactive consumers with or without ESSs, electricity prosumers with or without ESSs, and non-responsive customers. In a distribution market, each consumer can bid a price-demand pair and each producer can offer a price-output pair to the market, based on their forecasting of generation output and the arrangement of electricity consumptions. A distribution market can be organized in a flexible way, which can be operated as half-hourly or hourly ahead market instead of a day-ahead market, in order to have more accurate forecasting results when participating in the trading. Once winning the auction, the consumer will purchase electricity from the distribution market at the market clearing price (MCP); otherwise, the consumer will need to purchase electricity at the incumbent retail price from the utility company or an electricity retailer. Similarly, a producer will sell electricity to the distribution market at the MCP if winning the auction; otherwise will have to sell electricity to the utility grid at the feed-in tariff or to an electricity retailer. Fig. 1 depicts the potential interactions between the DMO and different market participants.

The major responsibilities of the DMO are as follows:

1) To receive demand bids and generation offers from market participants with or without ESSs;

2) To solve the market clearing problem and send market clearing outcomes to participants;

3) To monitor the implementation of trading deals and carry out ex-post settlement for participants.

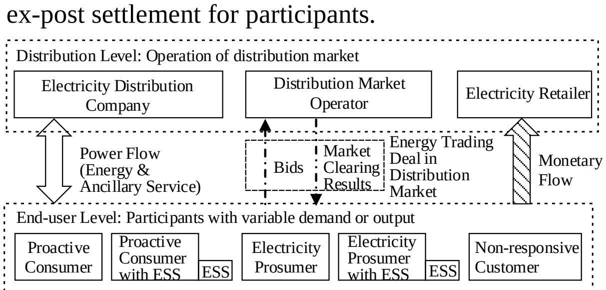

Fig.1 Interactions between the DMO and market participants

In order for the distribution market to work reliably, it is necessary that participants schedule their resources based on the assigned clearing outcomes, considering that they seek the maximization of their own benefits. Additional measures, such as a penalty scheme on deviations, should be introduced to make sure that the participants closely follow their energy trading deals in the distribution market. Besides, for those customers who do not want to participate in the distribution market, they can purchase electricity from the utility company or electricity retailers.

B. *Extension of the APM Mechanism*

An APM mechanism for clearing the market bidding of renewable generation with zero marginal costs in the distribution network is proposed by [11]. In the double-sided bidding mechanism, the i^{th} ($i \in N$) buyer submits a bid that contains a price-demand pair (r_i^{b}, p_i^{b}), where r_i^{b} and p_i^{b} indicate the price and demand, respectively. Meanwhile, the j^{th} ($j \in M$) seller offers a price-output pair (r_j^{s}, p_j^{s}) to the market. The weighted average $\bar{r}$ of participants' bid prices is adopted as the MCP, where the weighting factors are their bid quantities.

$$\bar{r} = \left(\sum_{i=1}^{N} r_i^{\text{b}} \cdot p_i^{\text{b}} + \sum_{j=1}^{M} r_j^{\text{s}} \cdot p_j^{\text{s}} \right) \Bigg/ \left(\sum_{i=1}^{N} p_i^{\text{b}} + \sum_{j=1}^{M} p_j^{\text{s}} \right) \tag{1}$$

Ref. [11] focuses on the analysis of bidding prices for participants and honesty is proved to be a dominant strategy. A strategy is called a dominant strategy if it maximizes the agent's expected utility for all possible strategies of other agents [16, 21]. Considering that re-bidding might be permitted in the proposed market, there will be two possible scenarios faced by participants: (1) if there is no re-bidding, all participants submit bids simultaneously and no market clearing information is available before hand; (2) if re-bidding is permitted, market clearing information such as the current clearing price and total trading volume will be released and participants will re-bid based on the known information. It is defined as honesty if a participant will bid at his/her self-estimated generation cost (for a producer) / electricity utility (for a consumer) when re-bidding is not permitted, and all participants bid and offer simultaneously. Otherwise, when re-bidding is permitted, it is defined as honesty when the bidding behaviour truly reflects the relationship between a participant's self-estimation of generation cost (for a producer) / electricity utility (for a consumer) and the MCP. In other words, being honest, a participant tends to submit a bid / an offer that is not less than/ not larger than the observed MCP if the self-estimated generation cost/electricity utility is indeed not less than/ not larger than the MCP.

Particularly, this paper further develops the proposed APM mechanism by analyzing the bidding quantities of demand/output and proved that the dominant bidding strategy for a participant is independent of the bidding quantity.

Theorem 1: *The dominant strategy of a participant under the APM mechanism is independent of the bidding quantity of demand/power generation.*

Proof: Apparently, if re-bidding is permitted, the MCP will change from $\bar{r}$ to $\bar{r}_{\text{new}}$ when consumer i re-bids at $r_i^{\text{b,new}}$. Let θ_i denote the percentage of the MCP change because of re-bidding, and p_{-i}^{b} denotes the bids of other participants except consumer i.

$$\bar{r}_{\text{new}} = (1 \pm \theta_i)\bar{r} = \frac{r_i^{\text{b,new}} p_i^{\text{b}} + \sum \bar{r} p_{-i}^{\text{b}}}{p_i^{\text{b}} + \sum p_{-i}^{\text{b}}} \Rightarrow \theta_i = \mp \left[\frac{p_i^{\text{b}} (r_i^{\text{b,new}} - \bar{r})}{\bar{r}(p_i^{\text{b}} + \sum p_{-i}^{\text{b}})} \right] \tag{2}$$

$$\theta_i = \frac{p_i^{\mathrm{b}}(r_i^{\mathrm{b,new}} - \bar{r})}{\bar{r}(p_i^{\mathrm{b}} + \sum p_{-i}^{\mathrm{b}})}, \text{if } r_i^{\mathrm{b,new}} > \bar{r}\ ;\ \theta_i = \frac{p_i^{\mathrm{b}}(\bar{r} - r_i^{\mathrm{b,new}})}{\bar{r}(p_i^{\mathrm{b}} + \sum p_{-i}^{\mathrm{b}})}, \text{if } r_i^{\mathrm{b,new}} < \bar{r} \quad (3)$$

Similarly, if let p_{-j}^{s} denote the offers of other participants except producer j, the MCP when producer j re-offers at $r_j^{\mathrm{s,new}}$ can be expressed as

$$\bar{r}_{\mathrm{new}} = (1 \pm \theta_j)\bar{r} = \frac{r_j^{\mathrm{s,new}} p_j^{\mathrm{s}} + \sum \bar{r} p_{-j}^{\mathrm{s}}}{p_j^{\mathrm{s}} + \sum p_{-j}^{\mathrm{s}}} \Rightarrow \theta_j = \pm\left[\frac{p_j^{\mathrm{s}}(r_j^{\mathrm{s,new}} - \bar{r})}{\bar{r}(p_j^{\mathrm{s}} + \sum p_{-j}^{\mathrm{s}})}\right] \quad (4)$$

$$\theta_j = \frac{p_j^{\mathrm{s}}(r_j^{\mathrm{s,new}} - \bar{r})}{\bar{r}(p_j^{\mathrm{s}} + \sum p_{-j}^{\mathrm{s}})}, \text{if } r_j^{\mathrm{s,new}} > \bar{r}\ ;\ \theta_j = \frac{p_j^{\mathrm{s}}(\bar{r} - r_j^{\mathrm{s,new}})}{\bar{r}(p_j^{\mathrm{s}} + \sum p_{-j}^{\mathrm{s}})}, \text{if } r_j^{\mathrm{s,new}} < \bar{r} \quad (5)$$

As can be observed in (3) and (5), θ_i and θ_j are determined by both the bidding prices (r_i^{b} and r_j^{s}) and quantities (p_i^{b} and p_j^{s}), given the bids of other participants. However, if the condition that consumers and producers are only allowed to submit a non-negative bid of load demand or power generation is specified, which means $p_i^{\mathrm{b}} \geq 0$ and $p_j^{\mathrm{s}} \geq 0$, then $\theta_i \geq 0$ and $\theta_j \geq 0$ would always hold.

Let b denote the utility of using electricity for consumer i.

(1) When $b > \bar{r}$, if consumer i re-bids at $r_i^{\mathrm{b,new}} > \bar{r}$, then the new MCP will be $\bar{r}_{\mathrm{new}}=(1+\theta_i)\bar{r}$ (where $\theta_i \geq 0$), the trading benefit will be b-$(1+\theta_i)\bar{r}$ which can be positive if $r_i^{\mathrm{b,new}}$ is properly chosen. If consumer i re-bids at $r_i^{\mathrm{b,new}} = \bar{r}$, then the new MCP will be $\bar{r}_{\mathrm{new}}=\bar{r}$, and the trading benefit will be b-$\bar{r}_{\mathrm{new}} = 0$; Otherwise, if consumer i re-bids at $r_i^{\mathrm{b,new}} < \bar{r}$, the new MCP will be $\bar{r}_{\mathrm{new}}=(1-\theta_i)\bar{r}$ (where $\theta_i \geq 0$), because $\bar{r}_{\mathrm{new}}$ is a weighted average of $r_i^{\mathrm{b,new}}$ and $\bar{r}$, so $r_i^{\mathrm{b,new}} < \bar{r}_{\mathrm{new}} < \bar{r}$, which means the re-bid price is smaller than the new MCP, namely consumer i loses the auction and has a trading benefit of 0. Thus, when $b > \bar{r}$, the best strategy for consumer i is to bid $r_i^{\mathrm{b}} > \bar{r}$ for a positive trading benefit of b-$(1+\theta_i)\bar{r}$.

(2) When $b = \bar{r}$, if consumer i re-bids at $r_i^{\mathrm{b,new}} > \bar{r}$, then the new MCP will be $\bar{r}_{\mathrm{new}}=(1+\theta_i)\bar{r}$ (where $\theta_i \geq 0$), and the trading benefit will be b-$(1+\theta_i)\bar{r} \leq 0$ since $\theta_i \geq 0$. If consumer i re-bids at $r_i^{\mathrm{b,new}} = \bar{r}$, the new MCP will be $\bar{r}_{\mathrm{new}}=\bar{r}$, and the trading benefit will be b-$\bar{r}_{\mathrm{new}} = 0$; However, if consumer i re-bids at $r_i^{\mathrm{b,new}} < \bar{r}$, the new MCP will be $\bar{r}_{\mathrm{new}}=(1-\theta_i)\bar{r}$ (where $\theta_i \geq 0$), because $\bar{r}_{\mathrm{new}}$ is a weighted average of $r_i^{\mathrm{b,new}}$ and $\bar{r}$, thus $r_i^{\mathrm{b,new}} < \bar{r}_{\mathrm{new}} < \bar{r}$, which means consumer i loses the auction and this results in a trading benefit of 0. Thus, when $b = \bar{r}$, the consumer i cannot do better than bidding at $r_i^{\mathrm{b}} = \bar{r}$ with a trading benefit of 0.

(3) When $b < \bar{r}$, if consumer i re-bids at $r_i^{\mathrm{b,new}} > \bar{r}$, then the new MCP will be $\bar{r}_{\mathrm{new}}=(1+\theta_i)\bar{r}$ (where $\theta_i \geq 0$), and the trading benefit will be b-$(1+\theta_i)\bar{r} \leq 0$ since $\theta_i \geq 0$. If consumer i re-bids at $r_i^{\mathrm{b,new}} = \bar{r}$, the new MCP will be $\bar{r}_{\mathrm{new}}=\bar{r}$, and the trading benefit will be b-$\bar{r}_{\mathrm{new}} = 0$; Otherwise, If consumer i re-bids at $r_i^{\mathrm{b,new}} < \bar{r}$, then the new MCP will be $\bar{r}_{\mathrm{new}}=(1-\theta_i)\bar{r}$ (where $\theta_i \geq 0$), since $\bar{r}_{\mathrm{new}}$ is a weighted average of $r_i^{\mathrm{b,new}}$ and $\bar{r}$, so $r_i^{\mathrm{b,new}} < \bar{r}_{\mathrm{new}} < \bar{r}$, namely the re-bid price is smaller than the new MCP, then consumer i loses the auction has a trading benefit of 0. Therefore, when $b < \bar{r}$, consumer i cannot do better than bidding at $r_i^{\mathrm{b}} < \bar{r}$ with a trading benefit of 0.

Likewise, let s denote the self-estimated generation cost for producer j. When $s < \bar{r}$, the best strategy for producer j is to offer at $r_j^{\mathrm{s}} < \bar{r}$ for a positive transaction benefit of $(1-\theta_j)\bar{r}$ -s. When $s = \bar{r}$ and $s > \bar{r}$, producer j cannot do better than offering at $r_j^{\mathrm{s}} = \bar{r}$ and $r_j^{\mathrm{s}} > \bar{r}$ with a transaction benefit of 0, because a non-positive benefit of $(1-\theta_j)\bar{r}$ -$s \leq 0$ will always be obtained ($\theta_j \geq 0$ for any bidding quantity of p_j^{s}).

In other words, when making decisions to maximize their own utilities, both a consumer and a producer cannot do better than bidding honestly in the proposed market mechanism. More importantly, the dominant strategy for a participant is independent of their bidding quantities of load demand/power generation. Hence, Theorem 1 is proved.

A key feature of an electricity distribution market is that it may be dominated by renewable generation with zero marginal cost. Existing market mechanisms are likely to fail in this context since it cannot generate a reasonable price signal to compensate for the investment cost of renewable generators. Consequently, the APM Mechanism which is proposed for pricing the zero marginal cost renewable generation in an electricity distribution market is adopted in this paper. More detailed elaboration about the advantages of APM is presented in [11].

Besides, current wholesale electricity markets around the globe have quite similar basic architectures, where their general function is to provide a non-discriminatory and transparent platform for transactions of electricity commodities. According to different time regimes, a wholesale electricity market can be generally categorized into forward electricity market (FEM), day-ahead market (DAM), intraday market (IDM), and real-time market (RTM) [22]. The APM is a mechanism mainly for pricing electricity energy commodity which can be transacted in the DAM, IDM or RTM.

As is observed in practices, participants in existing electricity markets can participate in both energy and ancillary service markets simultaneously, bidding strategies of participants in the energy market where the APM applies can be impacted by transactions in the ancillary service market. Besides, due to the large variety of ancillary service commodities in a power system, transactions of ancillary services can span the FEM, DAM, IDM and RTM. However, this paper also proves that the dominant strategy of participants under the APM mechanism is independent of the transactions in ancillary service markets. Thus, merits of the APM will be retained when it is adopted for a joint participation in energy and ancillary service markets by a participant, since the bidding strategies of participants in the energy market will not be impacted by the cost of purchasing FRS from ESSs.

Due to the mentioned transaction features in the distribution market, the APM mechanism differs from existing marginal cost based power pool, as implemented in the Australian national electricity market (NEM) and most deregulated electricity markets in USA, and the bilateral contract model in the UK electricity market in the following aspects. First, as most existing power pools are designed based on the marginal cost and marginal revenue theory, in the scenario with 100% renewable generation bidding, these market mechanisms will fail to price the renewable energy generation properly. The APM is designed to solve this problem. Secondly, in the APM, a dominant bidding strategy is proved to exist in the proposed market mechanism, which enables the APM to develop into a set-and-forget bidding market which cannot be attained under other market mechanisms. In particular, an intensive involvement of participants is needed in the bilateral contract market since both transaction quantities and prices are determined through bilateral negotiation.

III. Proposed Penalty Scheme on Deviation of Real-Time Demand and Output

A. Decision-making of Participants in a Distribution Market

In the distribution market, all winning participants are required to comply with market clearing outcomes. Otherwise, a penalty will apply to the deviation between their real-time demand/output and their winning bids/offers. Due to the variability of renewable generation and inherent randomness of end-users' electricity consumption, both sellers and buyers will need to utilized their self-equipped ESS or to purchase FRS from other ESSs to compensate for the real-time imbalance.

Ref. [23] reviews the settlement rule of regulation services in several representative electricity markets in North America. It is pointed out that with the emergence of fast-response resources, e.g. battery energy storage systems (BESSs), ISOs are required by the Federal Energy Regulatory Commission (FERC) to differentiate payment for FRS using a two-part compensation scheme: capacity and mileage settlements [24]. The capacity price considers the opportunity cost of offering FRS, while the mileage price reflects the actual amount of up and down regulation during providing the service.

If there is no penalty applied to the deviation between demand and power generation, the DMO is unable to maintain the generation-load balance; this would result in extra cost for maintaining the steady-state frequency of power systems. Let Δt denote the interval of transaction in the distribution market. Fig.2 illustrates different time regimes for transactions in the distribution market and the real-time demand/output deviations.

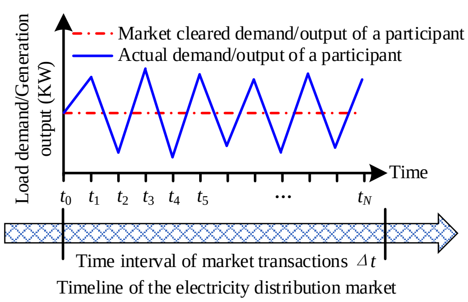

Fig.2 Time regimes of distribution market trading and real-time demand/output

Notably, due to widely installations of smart meters, load/output data of prosumers in a distribution system can be recorded at a high frequency, such as at a second interval or more frequently, for monitoring and billing purposes. In the proposed penalty scheme, the deviation between demand and power generation is considered to be penalized by the maximum deviation of power rate, the regulation mileage, and the energy difference during Δt. However, participants can initiatively manage the fluctuations of output/demand by allocating the usage of their own ESSs or purchasing beforehand a certain amount of ESS capacity for FRS from the regulation market. This regulation market is also a distribution level market where small-scale ESS owners or aggregators can trade with distribution energy market participants for ESS FRS, and is operated after the distribution energy market. Thus, it is essential to guide participants with potential positive benefits to participate in the energy market. Otherwise, the actual deviation of output/demand could be severe. This can be carried out by releasing parameters that are defined in the proposed penelaty scheme to energy market participants.

Notably, due to the time-varying MCP in a distribution market, participants have opportunities to arbitrage by buying energy at a low price and selling it at a higher price using their self-equipped ESS. Meanwhile, the MCP of a distribution market has a price floor of the feed-in tariff r^{feed}, and is capped by the incumbent electricity retail price r^{retail}. Thus, a participant needs to optimize the allocation of ESS capacity among arbitrage, self-regulation, and selling FRS to others. The decision-making problem faced by a buyer with self-equipped ESS in a distribution market can be modeled as follows.

$$\max\ R_i^{\text{b}} = R_i^{\text{b,ESS,rsv}} + R_i^{\text{b,ESS,mkt}} + R_i^{\text{b,ret}} - C_i^{\text{b,ESS,FRS}} - C_i^{\text{b,pnl,c}} - C_i^{\text{b,pnl,mil}} - C_i^{\text{b,pnl,e}} \tag{6}$$

$$R_i^{\text{b,ESS,rsv}} = p_i^{\text{b,ESS,rsv}} (r^{\text{retail}} - r^{\text{feed}}) \Delta t \tag{7}$$

$$R_i^{\text{b,ESS,mkt}} = p_i^{\text{b,ESS,mkt}} r^{\text{c,ESS}} \tag{8}$$

$$p_i^{\text{b,ESS}} = p_i^{\text{b,ESS,rsv}} + p_i^{\text{b,ESS,mkt}} + p_i^{\text{b,ESS,reg}} \tag{9}$$

$$R_i^{\text{b,ret}} = p_i^{\text{cb}} (r^{\text{retail}} - \bar{r}) \Delta t \tag{10}$$

$$C_i^{\text{b,ESS,FRS}} = p_i^{\text{b,ESS,c}} r^{\text{c,ESS}} + r^{\text{mil}} \sum_{s \in \Delta t} \left| p_{i,s+1}^{\text{b,acl}} - p_{i,s}^{\text{b,acl}} \right| \tag{11}$$

$$C_i^{\text{b,pnl,c}} = r^{\text{b,pnl,c}} \max_{s \in \Delta t} \left(\left| p_i^{\text{cb}} - p_{i,s}^{\text{b,acl}} \right| - p_i^{\text{b,ESS,c}} - p_i^{\text{b,ESS,reg}}, 0 \right) \tag{12}$$

$$C_i^{\text{b,pnl,mil}} = r^{\text{b,pnl,mil}} \sum_{s \in \Delta t} \max \left[\left| p_{i,s+1}^{\text{b,acl}} - p_{i,s}^{\text{b,acl}} \right| - 2(p_i^{\text{b,ESS,c}} + p_i^{\text{b,ESS,reg}}), 0 \right] \tag{13}$$

$$C_i^{\text{b,pnl,e}} = r^{\text{b,pnl,e}} \left[\left| \int_{s \in \Delta t} (p_{i,s}^{\text{b,acl}} - p_i^{\text{cb}}) ds \right| - \int_{s \in \Delta t} p_i^{\text{b,ESS,reg}} ds \right]^+ \tag{14}$$

$$s.t.\quad 0 \le p_i^{\text{b,ESS,rsv}}, p_i^{\text{b,ESS,mkt}}, p_i^{\text{b,ESS,reg}};\ 0 \le p_i^{\text{b,ESS,c}} \le p^{\text{ESS,c,avil}} \tag{15}$$

where R_i^{b} is the total benefit of the i^{th} buyer; $R_i^{\text{b,ESS,rsv}}$ and $R_i^{\text{b,ESS,mkt}}$ are the revenue of arbitrage and selling FRS to others, respectively; $R_i^{\text{b,ret}}$ denotes the saving of money by participating in the distribution market; $C_i^{\text{b,ESS,FRS}}$ represents the cost of purchasing ESS capacity and regulation mileage; $C_i^{\text{b,pnl,c}}/C_i^{\text{b,pnl,mil}}/C_i^{\text{b,pnl,e}}$ represents the penalty on actual maximum electricity power rate deviation/FRS regulation mileage/difference of electricity quantity between the real-time and market cleared load electricity consumptions; $p_i^{\text{b,ESS}}$ indicates the capacity of self-equipped ESS; $p_i^{\text{b,ESS,rsv}}/p_i^{\text{b,ESS,mkt}}/p_i^{\text{b,ESS,reg}}$ is the allocated ESS capacity for arbitrage/selling FRS to others/self-regulation; p_i^{cb} is the winning load demand; $p_i^{\text{b,ESS,c}}$ is the purchased ESS capacity for FRS; $r^{\text{c,ESS}}$ and r^{mil} are the capacity and mileage prices of ESS FRS, respectively; $p_{i,t}^{\text{b,acl}}$ denotes the actual load demand of buyer i at time t; $r^{\text{b,pnl,c}}$, $r^{\text{b,pnl,mil}}$ and $r^{\text{b,pnl,e}}$ denote the penalty prices of electricity power rate deviation, regulation mileage, and difference of electricity quantity between the real-time and market cleared load electricity consumptions, respectively; $p^{\text{ESS,c,avil}}$ is the available ESS capacity for purchasing. For buyer i without its own ESS, the problem can be simplified to the case of $p_i^{\text{b,ESS}}=0$.

In the decision-making problem of a buyer in a distribution market as modelled by (6)-(15), decision variables are $p_i^{\text{b,ESS,rsv}}$, $p_i^{\text{b,ESS,mkt}}$, $p_i^{\text{b,ESS,reg}}$ and $p_i^{\text{b,ESS,c}}$. Meanwhile, p_i^{cb} is the decision variable in the previous distribution market clearing mechanism and is already known at this stage. Once the values of parameters in (6)-(15) are given, which represent the distribution market conditions, the buyer will make optimal decisions to maximize its overall profit.

The same penalty scheme also applies to all sellers. Each seller with a self-equipped ESS will maximize its benefit in the distribution market by solving the following problem.

$$\max R_j^{s} = R_j^{s,ESS,rsv} + R_j^{s,ESS,mkt} + R_j^{s,ret} - C_j^{s,ESS,FRS} - C_j^{s,pnl,c} - C_j^{s,pnl,mil} - C_j^{s,pnl,e} \quad (16)$$

$$R_j^{s,ESS,rsv} = p_j^{s,ESS,rsv}(r^{retail} - r^{feed})\Delta t \quad (17)$$

$$R_j^{s,ESS,mkt} = p_j^{s,ESS,mkt} r^{c,ESS} \quad (18)$$

$$p_j^{s,ESS} = p_j^{s,ESS,rsv} + p_j^{s,ESS,mkt} + p_j^{s,ESS,reg} \quad (19)$$

$$R_j^{s,ret} = p_j^{cs}(\bar{r} - r^{feed})\Delta t \quad (20)$$

$$C_j^{s,ESS,FRS} = p_j^{s,ESS,c} r^{c,ESS} + r^{mil} \sum_{s\in\Delta t} \left| p_{j,s+1}^{s,acl} - p_{j,s}^{s,acl} \right| \quad (21)$$

$$C_j^{s,pnl,c} = r^{s,pnl,c} \max_{s\in\Delta t}\left(\left| p_j^{cs} - p_{j,s}^{s,acl} \right| - p_j^{s,ESS,c} - p_j^{s,ESS,reg}, 0\right) \quad (22)$$

$$C_j^{s,pnl,mil} = r^{s,pnl,mil} \sum_{s\in\Delta t} \max\left[\left| p_{j,s+1}^{s,acl} - p_{j,s}^{s,acl} \right| - 2(p_j^{s,ESS,c} + p_j^{s,ESS,reg}), 0\right] \quad (23)$$

$$C_j^{s,pnl,e} = r^{s,pnl,e}\left[\left| \int_{s\in\Delta t} (p_{j,s}^{s,acl} - p_j^{cs}) ds \right| - \int_{s\in\Delta t} p_j^{s,ESS,reg} ds\right]^{+} \quad (24)$$

$$s.t.\quad 0 \le p_j^{s,ESS,rsv}, p_j^{s,ESS,mkt}, p_j^{s,ESS,reg};\ 0 \le p_j^{s,ESS,c} \le p^{ESS,c,avil} \quad (25)$$

where $p_j^{s,ESS}$ denotes the capacity of a self-equipped ESS for the j^{th} producer; $p_j^{s,ESS,rsv}/p_j^{s,ESS,mkt}/p_j^{s,ESS,reg}$ is the allocated ESS capacity for arbitrage/selling FRS to others/self-regulation; p_j^{cs} is the winning power generation of the j^{th} producer; $p_j^{s,ESS,c}$ is the purchased ESS capacity for FRS; $p_{j,t}^{s,acl}$ is the actual output of seller j at time t; $r^{s,pnl,c}$, $r^{s,pnl,mil}$ and $r^{s,pnl,e}$ are the penalty prices on electricity power rate deviation, regulation mileage, and difference of electricity quantity between the real-time and market cleared generation outputs, respectively. Besides, the meaning of each item in (16) is similar to that in (6). For sellers without their own ESS, the problem can be simplified to the case of $p_i^{s,ESS}=0$.

Similarly, in the decision-making problem of (16)-(25), decision variables are $p_j^{s,ESS,rsv}$, $p_j^{s,ESS,mkt}$, $p_j^{s,ESS,reg}$ and $p_j^{s,ESS,c}$. Moreover, p_j^{cs} is determined in the previous distribution market clearing process and is known at this stage. Given values of parameters in (16)-(25), which represent the distribution market conditions, the seller will try to develop optimal strategies to maximize its overall benefit.

When an ESS is used for frequency regulation and black start, it is usually measured by the capacity of power electronic converter in kW or MW. When the ESS is used for other applications including renewable generation integration, peak shaving, and load levelling, the ESS is usually measured by its power storage capacity in kWh or MWh. Besides, the state-of-charge (SOC) indicates the level of charge of an ESS relative to its power storage capacity, which is a measure of the remaining energy in the ESS. In this paper, it is the ESS FRS that is considered and studied in an electricity distribution market, as is modelled by (6)-(15) and (16)-(25), the ESS capacity refers to the capacity of power electronic converter instead of the energy storage. Furthermore, considering that the temporal resolution of FRS is very high, the SOC constraints of ESSs are therefore not considered.

The presented methodological framework as formulated in (6)-(15) and (16)-(25) can accommodate SOC constraints, and the decision variables about the allocation of self-equipped ESS capacity can then be binded. However, including SOC constraints does not affect the analysis on the setting of penalty prices in order to mitigate uninstructed deviations, which is the focus of this paper. Consequently, the SOC constraints of ESSs are not included.

In the decision-making problems for both buyers and sellers, physical constraints such as three-phase power flow, operating reserves, nodal voltages, and network loss are not included. First, this is because the distribution market mechanism studied is for active power transactions. All reactive power involved problems are not involved, since the pricing of reactive power in a distribution network is another sophisticated problem. Secondly, the focus of this paper is to derive the setting of penalty prices through the revenue and cost analysis for distribution market participants, in order to guide these participants to actively compensate the uninstructed deviation of load demand or generation output. Although the integration of more physical constraints in the decision-making problem may help derive more accurate decisions for participants, these constraints do not contribute to the derivation of penalty prices, and then these constraints are not included for more concise analysis. Besides, since the proposed penalty scheme is to ensure accurate implementation of market clearing outcomes, but not for the secure operation of the concerned distribution system, there is no need to include the operating reserve constraint.

B. Penalty Price of Real-Time Imbalance

For each participant in the distribution market, only when the penalty is higher than the potential arbitrage benefit, the cost of purchasing FRS and the revenue of selling their own ESS FRS to others, then it would initiatively choose to use either its own ESS or purchased FRS provided by an ESS for mitigating demand/output deviations to avoid penalty. Therefore, for the i^{th} buyer, the following inequality will hold under any realization of $p_{i,t}^{b,acl}$.

$$r^{b,pnl,e} \ge r^{retail} - r^{feed},\ r^{b,pnl,c} \ge r^{c,ESS}, \text{ and } r^{b,pnl,mil} \ge r^{mil} \quad (26)$$

Eqn. (26) indicates that the i^{th} buyer will always be subject to a higher loss if it chooses to accept penalty rather than initiatively conducts self-regulation or purchasing FRS provided by an ESS. Similarly, the following inequality will hold under any realization of $p_{j,t}^{s,acl}$.

$$r^{s,pnl,e} \ge r^{retail} - r^{feed},\ r^{s,pnl,c} \ge r^{c,ESS}, \text{ and } r^{s,pnl,mil} \ge r^{mil} \quad (27)$$

Consequently, in order to motivate participates to initiatively compensate deviations, critical conditions on the prices of penalty and FRS provided by an ESS are presented as follows.

$$\min(r^{b,pnl,e}, r^{s,pnl,e}) \ge r^{retail} - r^{feed} \quad (28)$$

$$\min(r^{b,pnl,c}, r^{s,pnl,c}) \ge r^{c,ESS} \quad (29)$$

$$\min(r^{b,pnl,mil}, r^{s,pnl,mil}) \ge r^{mil} \quad (30)$$

Under the proposed penalty scheme, it can be further proved that the extent of an unexpected deviation can be controlled within a certain limit when the penalty prices are set properly.

Theorem 2: *For the i^{th} buyer and j^{th} seller, when the penalty prices on electricity power rate deviation, regulation mileage, and difference of electricity quantity between the real-time and market cleared outcomes are set as (31), (32) and (33), the maximum electricity power rate deviation and regulation mileage that exceed the purchased ESS capacity, and the ac-*

cumulated energy imbalance can be controlled within $L^{dev,max}p_i^{cb}/L^{dev,max}p_j^{cs}$, $M^{b,mil}/M^{s,mil}$, *and* $L^{e,dev,max}p_i^{cb}\Delta t/L^{e,dev,max}p_j^{cs}\Delta t$, *respectively:*

$$r^{\text{b,pnl,c}}=(r^{\text{retail}}-\bar{r})\Delta t\big/L^{\text{dev,max}}\;;\;r^{\text{s,pnl,c}}=(\bar{r}-r^{\text{feed}})\Delta t\big/L^{\text{dev,max}} \tag{31}$$

$$r^{\text{b,pnl,mil}}=\frac{(r^{\text{retail}}-\bar{r})\Delta t}{2N^{\text{disp}}L^{\text{b,dev,eql}}}\;;\;r^{\text{s,pnl,mil}}=\frac{(\bar{r}-r^{\text{feed}})\Delta t}{2N^{\text{disp}}L^{\text{s,dev,eql}}} \tag{32}$$

$$r^{\text{b,pnl,e}}=(r^{\text{retail}}-\bar{r})\big/L^{\text{e,dev,max}}\;;\;r^{\text{s,pnl,e}}=(\bar{r}-r^{\text{feed}})\big/L^{\text{e,dev,max}} \tag{33}$$

$$M^{\text{b,mil}}=2N^{\text{disp}}L^{\text{b,dev,eql}}p_i^{\text{cb}}\;;\;M^{\text{s,mil}}=2N^{\text{disp}}L^{\text{s,dev,eql}}p_j^{\text{cs}} \tag{34}$$

where $L^{\text{dev,max}}$ is the maximum percentage of acceptable electricity power rate deviation over the winning bids; $L^{\text{b,dev,eql}}/L^{\text{s,dev,eql}}$ denote the percentage of the deviation mileage over the winning bids for real-time load / generation output; when there is a difference of electricity quantity between the real-time and market cleared outcomes, $L^{\text{e,dev,max}}$ is the percentage of this difference over the winning bids; $M^{\text{b,mil}}$ / $M^{\text{s,mil}}$ is the total mileage of deviation for a buyer / seller; N^{disp} is the number of recorded demand/output data during Δt; p_i^{cb} and p_j^{cs} indicate the winning demand and output for the i^{th} consumer and j^{th} producer, respectively.

The proposed penalty scheme and concepts in Theorem 2 are illustrated in Fig.3. In Fig.3, the equivalent fluctuation means that for any $M^{\text{b,mil}}/M^{\text{s,mil}}$, there always exists $L^{\text{b,dev,eql}}/L^{\text{s,dev,eql}}$ that makes (34) hold, namely any deviation with a mileage $M^{\text{b,mil}}/M^{\text{s,mil}}$ during Δt can be mathematically equivalent to a fluctuation process with a time-invariant amplitude during Δt.

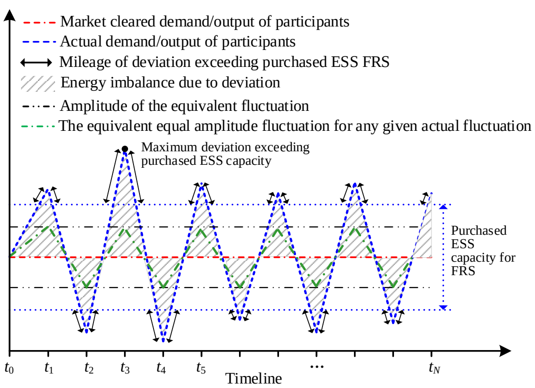

Fig.3 Schematic diagram of the concepts in Theorem 2

Proof: As modelled by (6)-(15), each participant makes decisions aiming at maximizing its overall profit. As mentined before, the regulation market usually opens after the energy market, so it is essential to guide participants with potential positive benefits to participate in the energy market. This can be carried out by releasing parameters $L^{\text{dev,max}}$, $L^{\text{b,dev,eql}}$, $L^{\text{s,dev,eql}}$ and $L^{\text{e,dev,max}}$ to energy market participants, which can also be viewed as technical requirements for market access.

First, it can be assumed that a participant does not purchase FRS by an ESS in the regulation market for compensating deviation. Since $L^{\text{dev,max}}$ denotes the maximum percentage of acceptable deviation over the winning bids, it means the final profit of the participant will be zero as long as the deviation reaches this limit. The acceptable deviation means that extra severe penalties will apply or participants will be excluded from the market if the deviation exceeds this acceptable limit:

When $\max\limits_{s\in\Delta t}(\left|p_i^{\text{cb}}-p_{i,s}^{\text{b,acl}}\right|-p_i^{\text{b,ESS,reg}},0)=L^{\text{dev,max}}p_i^{\text{cb}}$ and $p_i^{\text{b,ESS,c}}=0$, then, $R_i^{\text{b}}=R_i^{\text{b,ESS,rsv}}+R_i^{\text{b,ESS,mkt}}+R_i^{\text{b,ret}}-C_i^{\text{b,ESS,FRS}}-$

$$r^{\text{b,pnl,c}}L^{\text{dev,max}}p_i^{\text{cb}}-C_i^{\text{b,pnl,mil}}-C_i^{\text{b,pnl,e}} \tag{35}$$

Since $R_i^{\text{b,ESS,rsv}}$, $R_i^{\text{b,ESS,mkt}}$, $C_i^{\text{b,ESS,FRS}}$, $C_i^{\text{b,pnl,mil}}$, $C_i^{\text{b,pnl,e}}\geq 0$, thus $R_i^{\text{b}}\geq 0$

$$\Rightarrow r^{\text{b,pnl,c}}L^{\text{dev,max}}p_i^{\text{cb}}\leq R_i^{\text{b,ESS,rsv}}+R_i^{\text{b,ESS,mkt}}+R_i^{\text{b,ret}}$$

$$\Rightarrow r^{\text{b,pnl,c}}\leq\frac{R_i^{\text{b,ESS,rsv}}+R_i^{\text{b,ESS,mkt}}+R_i^{\text{b,ret}}}{L^{\text{dev,max}}p_i^{\text{cb}}}\leq\frac{(r^{\text{retail}}-\bar{r})\Delta t}{L^{\text{dev,max}}} \tag{36}$$

When $\max\limits_{s\in\Delta t}(\left|p_j^{\text{cs}}-p_{j,s}^{\text{s,acl}}\right|-p_j^{\text{s,ESS,reg}},0)=L^{\text{dev,max}}p_j^{\text{cs}}$ and $p_j^{\text{s,ESS,c}}=0$, then, $R_j^{\text{s}}=R_j^{\text{s,ESS,rsv}}+R_j^{\text{s,ESS,mkt}}+R_j^{\text{s,ret}}-$

$$C_j^{\text{s,ESS,FRS}}-r^{\text{s,pnl,c}}L^{\text{dev,max}}p_j^{\text{cs}}-C_j^{\text{s,pnl,mil}}-C_j^{\text{s,pnl,e}} \tag{37}$$

Since $R_j^{\text{s,ESS,rsv}}$, $R_j^{\text{s,ESS,mkt}}$, $C_j^{\text{s,ESS,FRS}}$, $C_j^{\text{s,pnl,mil}}$, $C_j^{\text{s,pnl,e}}\geq 0$, thus $R_j^{\text{s}}\geq 0$

$$\Rightarrow r^{\text{s,pnl,c}}L^{\text{dev,max}}p_j^{\text{cs}}\leq R_j^{\text{s,ESS,rsv}}+R_j^{\text{s,ESS,mkt}}+R_j^{\text{s,ret}}$$

$$\Rightarrow r^{\text{s,pnl,c}}\leq\frac{R_j^{\text{s,ESS,rsv}}+R_j^{\text{s,ESS,mkt}}+R_j^{\text{s,ret}}}{L^{\text{dev,max}}p_j^{\text{cs}}}\leq\frac{(\bar{r}-r^{\text{feed}})\Delta t}{L^{\text{dev,max}}} \tag{38}$$

Consequently, when the penalty prices $r^{\text{b,pnl,c}}$ and $r^{\text{s,pnl,c}}$ are set as their upper limits, as given by (31), the maximum percentage of acceptable deviation over the winning bids can be controlled under $L^{\text{dev,max}}$ in order for attaining positive $R_i^{\text{b}}/R_j^{\text{s}}$.

Otherwise, if a participant chooses to buy ESS capacity for compensating the maximum power deviation, then R_i^{b} and R_j^{s} would become positive values again because the cost of purchasing an unit of FRS provided by an ESS is smaller than the corresponding penalty. Besides, if the electricity power rate deviation exceeds the purchased ESS capacity, R_i^{b} and R_j^{s} would decrease to zero before the exceeded power deviation reaches $L^{\text{dev,max}}p_i^{\text{cb}}/L^{\text{dev,max}}p_j^{\text{cs}}$ as the purchased ESS capacity already incurred costs.

Under this circumstance, the maximum deviation of the power rate that is not covered by the purchased ESS capacity is limited under $L^{\text{dev,max}}p_i^{\text{cb}}/L^{\text{dev,max}}p_j^{\text{cs}}$.

Similarly, let $L^{\text{e,dev,max}}$ denote the upper limit of the acceptable energy deviation, which means the final profit of the participant will be zero as long as the deviation reaches this limit:

When $\left[\left|\int_{s\in\Delta t}(p_{i,s}^{\text{b,acl}}-p_i^{\text{cb}})ds\right|-\int_{s\in\Delta t}p_i^{\text{b,ESS,reg}}ds\right]^+=L^{\text{e,dev,max}}p_i^{\text{cb}}\Delta t$, and $p_i^{\text{b,ESS,c}}=0$, then

$$\begin{aligned}R_i^{\text{b}}=&R_i^{\text{b,ESS,rsv}}+R_i^{\text{b,ESS,mkt}}+R_i^{\text{b,ret}}-C_i^{\text{b,ESS,FRS}}-\\&C_i^{\text{b,pnl,c}}-C_i^{\text{b,pnl,mil}}-r^{\text{b,pnl,e}}L^{\text{e,dev,max}}p_i^{\text{cb}}\Delta t\end{aligned} \tag{39}$$

Since $R_i^{\text{b,ESS,rsv}}$, $R_i^{\text{b,ESS,mkt}}$, $C_i^{\text{b,ESS,FRS}}$, $C_i^{\text{b,pnl,c}}$, $C_i^{\text{b,pnl,mil}}\geq 0$, thus $R_i^{\text{b}}\geq 0$

$$\Rightarrow r^{\text{b,pnl,e}}L^{\text{e,dev,max}}p_i^{\text{cb}}\Delta t\leq R_i^{\text{b,ESS,rsv}}+R_i^{\text{b,ESS,mkt}}+R_i^{\text{b,ret}}$$

$$\Rightarrow r^{\text{b,pnl,e}}\leq\frac{R_i^{\text{b,ESS,rsv}}+R_i^{\text{b,ESS,mkt}}+R_i^{\text{b,ret}}}{L^{\text{e,dev,max}}p_i^{\text{cb}}\Delta t}\leq\frac{r^{\text{retail}}-\bar{r}}{L^{\text{e,dev,max}}} \tag{40}$$

When $\left[\left|\int_{s\in\Delta t}(p_{j,s}^{\text{s,acl}}-p_j^{\text{cs}})ds\right|-\int_{s\in\Delta t}p_j^{\text{s,ESS,reg}}ds\right]^+=L^{\text{e,dev,max}}p_j^{\text{cs}}\Delta t$, and $p_j^{\text{s,ESS,c}}=0$, then

$$\begin{aligned}R_j^{\text{s}}=&R_j^{\text{s,ESS,rsv}}+R_j^{\text{s,ESS,mkt}}+R_j^{\text{s,ret}}-C_j^{\text{s,ESS,FRS}}-\\&C_j^{\text{s,pnl,c}}-C_j^{\text{s,pnl,mil}}-r^{\text{s,pnl,e}}L^{\text{e,dev,max}}p_j^{\text{cs}}\Delta t\end{aligned} \tag{41}$$

Since $R_j^{\text{s,ESS,rsv}}$, $R_j^{\text{s,ESS,mkt}}$, $C_j^{\text{s,ESS,FRS}}$, $C_j^{\text{s,pnl,c}}$, $C_j^{\text{s,pnl,mil}}\geq 0$, thus $R_j^{\text{s}}\geq 0$

$$\Rightarrow r^{\text{s,pnl,e}}L^{\text{e,dev,max}}p_j^{\text{cs}}\Delta t\leq R_j^{\text{s,ESS,rsv}}+R_j^{\text{s,ESS,mkt}}+R_j^{\text{s,ret}}$$

$$\Rightarrow r^{\text{s,pnl,e}}\leq\frac{R_j^{\text{s,ESS,rsv}}+R_j^{\text{s,ESS,mkt}}+R_j^{\text{s,ret}}}{L^{\text{e,dev,max}}p_j^{\text{cs}}\Delta t}\leq\frac{\bar{r}-r^{\text{feed}}}{L^{\text{e,dev,max}}} \tag{42}$$

Then, the penalty prices $r^{\text{b,pnl,e}}$ and $r^{\text{s,pnl,e}}$ can be derived as (33). Given the above conditions, as long as the accumulated energy deviation exceeds $L^{\text{e,dev,max}} p_i^{\text{cb}} \Delta t$ and $L^{\text{e,dev,max}} p_j^{\text{cs}} \Delta t$, then R_i^{b} and R_j^{s} would decrease to zero.

In terms of regulation mileage, let $M^{\text{b,mil}}/M^{\text{s,mil}}$ indicates the upper limit. Similarly, the final profit of the participant is considered to be zero as long as the mileage reaches this limit. Meanwhile, the worst case corresponds to the situation when a participant does not purchase FRS provided by an ESS from the regulation market:

When $\sum_{s\in\Delta t}\max(\left|p_{i,s+1}^{\text{b,acl}}-p_{i,s}^{\text{b,acl}}\right|-2p_i^{\text{b,ESS,reg}},0)=M^{\text{b,mil}}$ and $p_i^{\text{b,ESS,c}}=0$,

then
$$R_i^{\text{b}}=R_i^{\text{b,ESS,rsv}}+R_i^{\text{b,ESS,mkt}}+R_i^{\text{b,ret}}-C_i^{\text{b,ESS,FRS}}-C_i^{\text{b,pnl,c}}-r^{\text{b,pnl,mil}}M^{\text{b,mil}}-C_i^{\text{b,pnl,e}} \tag{43}$$

Since $R_i^{\text{b,ESS,rsv}}$, $R_i^{\text{b,ESS,mkt}}$, $C_i^{\text{b,ESS,FRS}}$, $C_i^{\text{b,pnl,c}}$, $C_i^{\text{b,pnl,e}}\geq 0$, thus $R_i^{\text{b}}\geq 0 \Rightarrow$

$$r^{\text{b,pnl,mil}}\leq\frac{R_i^{\text{b,ESS,rsv}}+R_i^{\text{b,ESS,mkt}}+R_i^{\text{b,ret}}}{M^{\text{b,mil}}}\leq\frac{p_i^{\text{cb}}(r^{\text{retail}}-\bar{r})\Delta t}{M^{\text{b,mil}}} \tag{44}$$

When $\sum_{s\in\Delta t}\max(\left|p_{j,s+1}^{\text{s,acl}}-p_{j,s}^{\text{s,acl}}\right|-2p_j^{\text{s,ESS,reg}},0)=M^{\text{s,mil}}$ and $p_j^{\text{s,ESS,c}}=0$,

then
$$R_j^{\text{s}}=R_j^{\text{s,ESS,rsv}}+R_j^{\text{s,ESS,mkt}}+R_j^{\text{s,ret}}-C_j^{\text{s,ESS,FRS}}-C_j^{\text{s,pnl,c}}-r^{\text{s,pnl,mil}}M^{\text{s,mil}}-C_j^{\text{s,pnl,e}} \tag{45}$$

Since $R_j^{\text{s,ESS,rsv}}$, $R_j^{\text{s,ESS,mkt}}$, $C_j^{\text{s,ESS,FRS}}$, $C_j^{\text{s,pnl,c}}$, $C_j^{\text{s,pnl,e}}\geq 0$, thus $R_j^{\text{s}}\geq 0 \Rightarrow$

$$r^{\text{s,pnl,mil}}\leq\frac{R_j^{\text{s,ESS,rsv}}+R_j^{\text{s,ESS,mkt}}+R_j^{\text{s,ret}}}{M^{\text{s,mil}}}\leq\frac{p_j^{\text{cs}}(\bar{r}-r^{\text{feed}})\Delta t}{M^{\text{s,mil}}} \tag{46}$$

Likewise, the participant can also choose to buy FRS provided by ESSto reduce the penelty on mileage. Then R_i^{b} and R_j^{s} would become positive values again since the cost of purchasing an unit FRS provided by an ESS is smaller than the corresponding penalty. If the mileage occurs beyond the purchased ESS capacity, then R_i^{b} and R_j^{s} would decrease to zero before the mileage (that is not covered by the ESS capacity) reaches M^{mil} because the purchased ESS mileage within $p_i^{\text{b,ESS,c}}$ and $p_j^{\text{s,ESS,c}}$ already incurred costs.

Furthermore, for any $M^{\text{b,mil}}/M^{\text{s,mil}}$, there always exists a $L^{\text{b,dev,eql}}/L^{\text{s,dev,eql}}$ that makes (34) hold. Eqn. (34) means that any deviation with a mileage $M^{\text{b,mil}}/M^{\text{s,mil}}$ during Δt is mathematically equivalent to a fluctuation with the same amplitude each time during Δt. By substituting (34) into (44) and (46), $r^{\text{b,pnl,mil}}$ and $r^{\text{s,pnl,mil}}$ can be derived by (32).

Thus, with above constraints on the penalty prices, the maximum mileage that is not covered by the purchased ESS capacity can be controlled within $M^{\text{b,mil}}/M^{\text{s,mil}}$. Therefore, Theorem 2 is proved.

C. Market Surplus Analysis

When implementing the proposed penalty scheme in the regulation market, the allowed maximum deviation $L^{\text{dev,max}}$ of demand/power generation in the distribution market should be first defined by the DMO after the available ESS capacity in the distribution system is estimated. The DMO would need to ensure that the available ESS capacity be sufficient to compensate the overall fluctuations.

$$\frac{\max_{s\in\Delta t}(\left|p_i^{\text{cb}}-p_{i,s}^{\text{b,acl}}\right|)}{p_i^{\text{cb}}}\leq L^{\text{dev,max}};\ \frac{\max_{s\in\Delta t}(\left|p_j^{\text{cs}}-p_{j,s}^{\text{s,acl}}\right|)}{p_j^{\text{cs}}}\leq L^{\text{dev,max}} \tag{47}$$

Then, the penalty prices on the deviation of the power rate can be determined by (31).

Next, the DMO needs to pre-determine the maximum acceptable mileage for participants, and can be done through assigning values to $L^{\text{b,dev,eql}}$ and $L^{\text{s,dev,eql}}$. Consequently, the penalty prices of the mileage can be determined by (32).

Theorem 3: *Under the proposed ESS-based ancillary service market, there is always non-negative market surplus.*

In the distribution market, each participant is required to pay the FRS fee based on its actual demand/output deviation. As mentioned before, the same mileage price r^{mil} will be applied to the ESS discharging and charging services for both consumers and producers. Considering the complementariness of participants in their demand and output fluctuations, it can be easily proved that a non-negative market surplus can be attained, and the proof is omitted here due to space limitation.

IV. Case Study and Discussions

A. Simulation Dataset

The IEEE 69-bus distribution system [25] is adopted to demonstrate the proposed method. Since the detailed modeling of the power flow problem is out of the scope of this paper, it is assumed that the distribution network operation is three-phase balanced. Based on actual residential solar data in an Australian distribution system presented in [26], the bid/offer quantities of participants are generated randomly but within the range between 1 and 5 kW. Besides, it is assumed that the offer prices of small-scale renewable generators fall within the range between 0.05 and 0.5 \$/kWh [27]. The distribution market is simulated in a 5-min interval and the time interval of the FRS signal is 5s.

B. Results and Discussions

In existing electricity markets, such as regional electricity markets in North America including CAISO (California Independent System Operator), MISO (Midcontinent Independent System Operator) and SPP (Southwest Power Pool), penalty schemes are applied to uninstructed deviations of electricity generation in the concerned power system. Notably, the uninstructed deviation penalty in these markets is only assessed based on the uninstructed imbalance energy caused by excessive or insufficient electricity generation beyond a tolerance band. This is different from the proposed approach in this paper where the maximum deviations of power rate, deviation mileage and accumulated imbalance energy are considered. The comparison of performance between the proposed penalty approach and the penalty scheme applied in Midcontinent electricity market is presented in Fig.4.

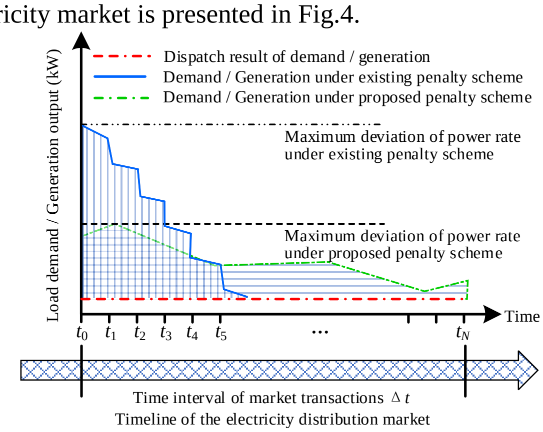

Fig.4 Comparison of performance between the existing penalty scheme in Midcontinent electricity market and the proposed approach

Fig.4 shows that load demand / generation output will be subject to different penalties under existing and proposed penalty schemes, although with the same quantity of accumulated imbalance energy (as is indicated by the shadow area in Fig.4). Under the proposed penalty scheme, each participant is also motived to achieve a smaller maximum deviation of the power rate and deviation mileage, in addition to trying to reduce the imbalance energy. In particular, the lower power deviation calls for less reserved regulation capacity in the concerned distribution system and a lower deviation mileage will help reduce the number of ESS recharge cycles and increase ESS lifespan. Therefore, compared with existing penalty mechanisms that mainly assess the penalty based on imbalance energy, the proposed approach takes into account the technical characteristics of ESSs in a more comprehensive way and is beneficial to the efficient operation of the distribution market.

Using the above specified data, simulations of the distribution market clearing process are carried out, and the attained market clearing price is 0.2780 \$/kWh. The incumbent electricity retail price and feed-in tariff is set as 0.5\$/kWh and 0.1 \$/kWh, respectively. The proposed market mechanism is tested given different maximum percentages of deviations $L^{\text{dev,max}}$, $L^{\text{b,dev,eql}}$ and $L^{\text{s,dev,eql}}$. Five different cases are considered and the penalty prices are calculated for these cases using (31)-(34) and presented in Tables I and II.

TABLE I Penalty Prices for Buyers under Five Cases

Case	$L^{\text{dev,max}}$	$L^{\text{b,dev,eql}}$	$L^{\text{e,dev,max}}$	Penalty prices $r^{\text{b,pnl,c}}$ (\$/kW)	$r^{\text{b,pnl,mil}}$ (\$/kW)	$r^{\text{b,pnl,e}}$ (\$/kWh)
1	50%	30%	30%	0.037	5.14e-4	0.74
2	40%	25%	25%	0.046	6.17e-4	0.89
3	30%	20%	20%	0.062	7.71e-4	1.11
4	20%	15%	15%	0.093	1.03e-3	1.48
5	10%	5%	5%	0.185	3.08e-3	4.44

TABLE II Penalty Prices for Sellers under Five Cases

Case	$L^{\text{dev,max}}$	$L^{\text{s,dev,eql}}$	$L^{\text{e,dev,max}}$	Penalty prices $r^{\text{s,pnl,c}}$ (\$/kW)	$r^{\text{s,pnl,mil}}$ (\$/kW)	$r^{\text{s,pnl,e}}$ (\$/kWh)
1	50%	30%	30%	0.030	4.12e-4	0.59
2	40%	25%	25%	0.037	4.94e-4	0.71
3	30%	20%	20%	0.049	6.18e-4	0.89
4	20%	15%	15%	0.074	8.24e-4	1.19
5	10%	5%	5%	0.148	2.47e-3	3.56

Results in Tables I and II show that even in the same case, different penalty prices can be applied to buyers and sellers in an electricity distribution market. This is because the benefits of participants are determined by the difference between the distribution market MCP and the incumbent retail price / feed-in tariff. Since in the proposed penalty scheme the penalty prices for participants are set by taking into account their concrete benefits from trading in the distribution market, and thus different penalty prices may be attained for various participants. Furthermore, Figs.5 and 6 present market clearing outcomes and the simulated deviation between load demand and generation output under case 1.

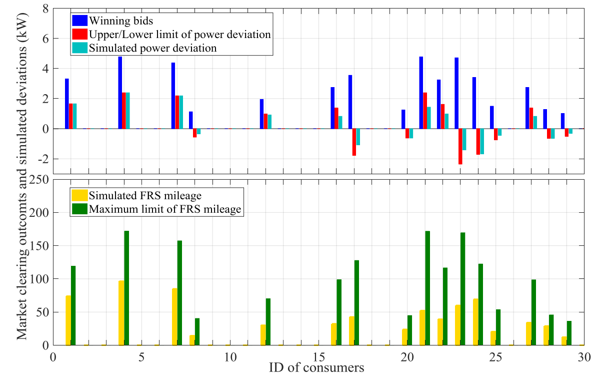

Fig.5 Distribution market clearing outcomes and simulated fluctuations of load demands by consumers

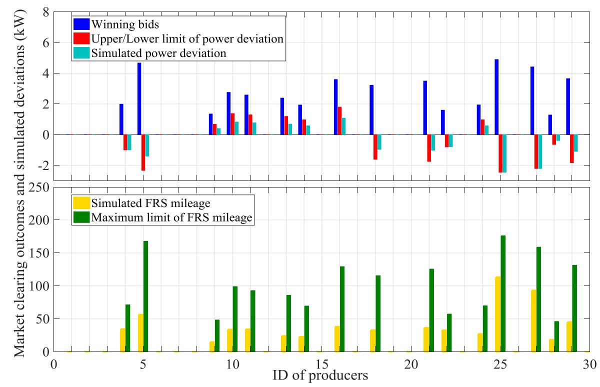

Fig.6 Distribution market clearing outcomes and simulated fluctuations of generation outputs from producers

From Figs.5 and 6, it can be observed that the absolute values of the maximum power deviations between demand and power generation are capped by $L^{\text{dev,max}}$. Meanwhile, fluctuation mileages of participants are subject to the upper limit as calculated by (33). Besides, Fig.7 shows the benefit obtained by each participant when the capacity price $r^{\text{c,ESS}}$ and mileage price r^{mil} of the FRS provided by ESSs are set as min ($r^{\text{b,pnl,c}}$, $r^{\text{s,pnl,c}}$) and min ($r^{\text{b,pnl,mil}}$, $r^{\text{s,pnl,mil}}$), respectively. In Fig.7, since the maximum power deviations of buyers (no. 1, 4, 7, 12, 20, 24, 28) and sellers (no. 4, 22, 25, 27) all reach the upper limits, as shown in Figs.6 and 7, the benefits of these buyers and sellers are therefore negative. On the contrary, all the other participants have smaller maximum power deviations, which help bring them positive benefits; this complies with the previous analysis associated with Theorem 2. Fig.7 also compares the benefits of participants with another case when $r^{\text{c,ESS}}$ is reduced by 0.01 \$/kW and results show that participants would attain higher benefits if the prices of FRS provided by ESSs reduce.

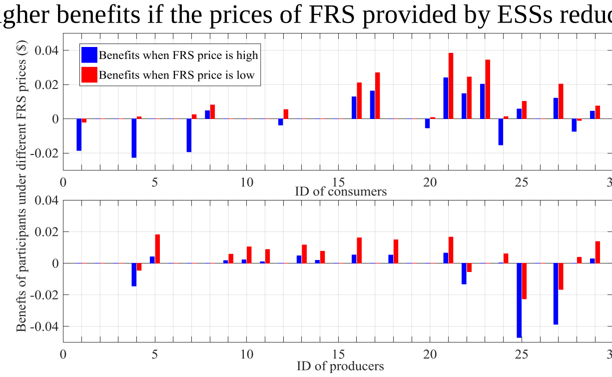

Fig.7 Benefits obtained by participants under different prices of ESS FRS

Sensitivity analysis for market surplus and market stability against penalty prices on power rate deviation, regulation mileage, and electricity quantity difference are also carried out. Three different scenarios are adopted: in scenario 1, $L^{\mathrm{dev,max}}$ decreases from 50% to 5% with $L^{\mathrm{b,dev,eql}}$, $L^{\mathrm{s,dev,eql}}$ and $L^{\mathrm{e,dev,max}}$ being fixed as 50%; in scenario 2, $L^{\mathrm{b,dev,eql}}$ and $L^{\mathrm{s,dev,eql}}$ decrease from 50% to 5% with $L^{\mathrm{dev,max}}$ and $L^{\mathrm{e,dev,max}}$ being fixed as 50% ; in scenario 3, $L^{\mathrm{e,dev,max}}$ decreases from 50% to 5% with $L^{\mathrm{dev,max}}$, $L^{\mathrm{b,dev,eql}}$ and $L^{\mathrm{s,dev,eql}}$ being fixed as 50%. Results under these three scenarios are presented in Tables III to V, respectively.

TABLE III CALCULATION RESULTS OF DIFFERENT CASES UNDER SCENARIO 1

Case	$L^{\mathrm{dev,max}}$	SumDM (kW)	SODM (kW)	SPmaxD (kW)	SplsDM (kW)
1	50%	1824.8	343.6997	13.1714	1481.1
2	45%	1645.1	311.4386	11.9244	1333.7
3	40%	1465.9	274.8913	10.4967	1191.0
4	35%	1286.9	242.3223	9.3265	1044.5
5	30%	1106.2	207.2575	8.0205	899.0
6	25%	925.2	174.7583	6.6789	750.5
7	20%	746.6	141.0486	5.36	605.6
8	15%	566.1	106.569	4.0529	459.6
9	10%	386.0	73.148	2.8113	312.8
10	5%	204.9	39.1767	1.4765	165.8

Note: SumDM denotes the sum of participants' deviation mileage; SODM represents the system overall deviation mileage; SPmaxD is the system maximum power deviation; SplsDM indicates the surplus of deviation mileage.

TABLE IV CALCULATION RESULTS OF DIFFERENT CASES UNDER SCENARIO 2

Case	$L^{\mathrm{b,dev,eql}}$ and $L^{\mathrm{s,dev,eql}}$	SumDM (kW)	SODM (kW)	SPmaxD (kW)	SplsDM (kW)
1	50%	1994.0	399.0272	16.8793	1595.0
2	45%	1869.8	372.8186	16.0057	1497.0
3	40%	1709.2	339.9214	15.2177	1369.3
4	35%	1551.2	309.3387	14.2089	1241.8
5	30%	1394.9	278.3714	13.2753	1116.6
6	25%	1239.1	247.3169	12.4845	991.8
7	20%	1077.6	218.4828	11.7324	859.1
8	15%	919.5	187.6154	11.0043	731.9
9	10%	714.4	150.5434	10.054	563.8
10	5%	363.5	77.35	8.1507	286.1

TABLE V CALCULATION RESULTS OF DIFFERENT CASES UNDER SCENARIO 3

Case	$L^{\mathrm{e,dev,max}}$	SumDM (kW)	SODM (kW)	SPmaxD (kW)	SplsDM (kW)
1	50%	1823.9	342.9407	13.0562	1481.0
2	45%	1825.8	344.2552	13.076	1481.6
3	40%	1826.2	342.4094	13.0652	1483.8
4	35%	1824.7	343.6705	13.0403	1481.0
5	30%	1824.6	344.1521	13.1117	1480.5
6	25%	1825.8	342.8401	13.1587	1483.0
7	20%	1824.2	343.4875	13.1593	1480.7
8	15%	1825.8	343.0843	13.1389	1482.7
9	10%	1826.0	346.0411	13.1333	1479.9
10	5%	1824.3	342.7691	13.0332	1481.5

Results in Tables III and IV show that from cases 1 to 10, with the decrease of $L^{\mathrm{dev,max}}$, $L^{\mathrm{b,dev,eql}}$ and $L^{\mathrm{s,dev,eql}}$, the FRS regulation mileage, the maximum power deviation, as well as the market surplus of mileage will also decrease. Meanwhile, as proved in Theorem 3, the surplus of the mileage is always non-negative. Different from $L^{\mathrm{dev,max}}$, $L^{\mathrm{b,dev,eql}}$ and $L^{\mathrm{s,dev,eql}}$, the change of $L^{\mathrm{e,dev,max}}$ can barely affect the maximum power deviation and total deviation mileage in a distribution market, as is shown in Table V. This is because the limits on the maximum power rate deviation, namely $L^{\mathrm{dev,max}}$, and limits on the total deviation mileage of the load demand / generation output, namely $L^{\mathrm{b,dev,eql}}$ / $L^{\mathrm{s,dev,eql}}$, can efficiently constrain fluctuations of demand / generation to a smaller range. However, the constraint on the electricity quantity difference, namely $L^{\mathrm{e,dev,max}}$, only focuses on the accumulated deviation for the whole period of time, and is unable to affect the fluctuation process, which finally leads to the above results.

Besides, Figs.8 and 9 present the relative changes of market surplus (measured by the surplus of mileage) and the relative changes of market stability (measured by the overall system deviation mileage) against the changes of penalty prices, respectively. It can be found that both the market surplus and stability have a similar sensitivity against the changes of penalty prices. Also, both the market surplus and stability are more sensitive to the changes of the penalty price on electricity power rate deviation than on regulation mileage and electricity quantity difference. This is because when the maximum power rate deviation is limited, both the potential regulation mileage and electricity quantity difference can also be affected. On the contrary, limitations on regulation mileage and electricity quantity difference will not be able to directly reduce the power rate deviation. Notably, the only changes of the penalty price on electricity quantity difference barely impact the distribution market surplus and stability. This complies with the previous analysis, since the penalty on energy difference is determined by the accumulated deviation over a certain time period but is unable to affect the fluctuation process.

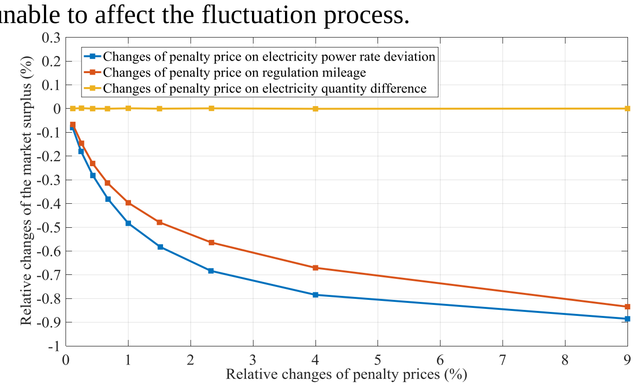

Fig.8 Sensitivity analysis of distribution market surplus against penalty prices

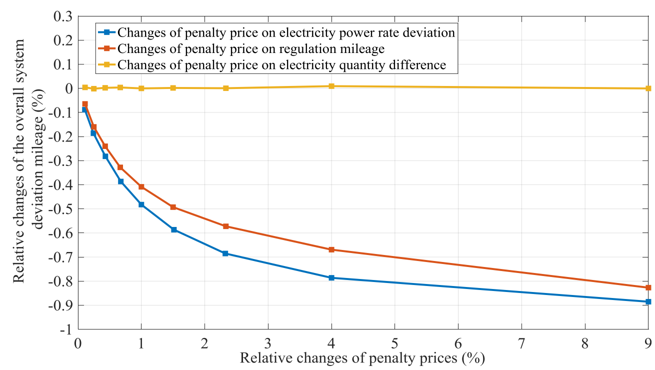

Fig.9 Sensitivity analysis of distribution market stability against penalty prices

To summarize, in designing an electricity distribution market, the special features of transactions should be properly taken into account, including the small trading volume of participants, the preference of set-and-forget bidding methods, as well as the zero marginal cost of distributed renewable generators. Otherwise, low efficiency or even operational failure of the distribution market could happen, such as the failure of existing marginal cost based electricity markets in pricing renewable generation. This is also one of the motivations for developing the APM mechanism in this paper.

In addition, the distribution market mechanism should also be compatible with uncertainties of electricity transactions, because of the high volatility of load demand and distributed renewable generation in a distribution network. First, it is suggested that in implementing the penalty scheme in practice, more efforts should be put on the determination of penalty prices on the maximum power rate deviation and regulation mileage rather than accumulated imbalance energy, because both the market surplus and stability are more sensitive to the changes of penalty prices on the electricity power rate deviation and regulation mileage than on the electricity quantity difference. Limitations on the maximum power rate deviation and regulation mileage can more efficiently mitigate the fluctuations of the load demand and generation output. Secondly, surveillance on the market surplus and stability is needed. In the proposed penalty scheme, a less severe penalty price can result in a higher market surplus but will lead to more severe market volatility. On the contrary, a more severe penalty can bring higher market stability but will reduce the market surplus. Therefore, a proper compromise between market surplus and market stability is necessary in setting the penalty prices.

V. Conclusions

Due to rapid growth of distributed renewable generation, the establishment of an appropriate electricity market mechanism for enhancing the accommodated capability for distributed renewable generation has gained world-wide concerns. However, there is not a sophisticated market mechanism available for efficiently settling electricity transactions in electricity distribution systems. Given this background, this paper further extends the previously developed APM to ensure accurate implementation of energy market clearing. Ancillary services from ESSs are considered and a penalty scheme established for managing volatile renewable energy generation outputs and end-user load demands. Case studies are carried out to demonstrate the feasibility and efficiency of the proposed models and algorithms. Meanwhile, critical suggestions about the operation of a distribution market are also provided based on simulation result analysis.

In the future, the interaction between the electricity wholesale market and the established distribution market will be studied. Another research subject is the design of ancillary service products provided by ESSs. With the ever growing distributed generation, more commercial opportunities will be available for ESSs; successful ESS service products can promote the capability of accommodating intermittent renewable energy generation in actual power systems.

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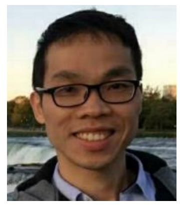

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